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Solving the inverse problem of arterial stiffness through Physics-Informed Neural Networks

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Abstract. Personalized 1D hemodynamic simulations can aid clinical decisions by modeling transient scenarios in real time. These models face one key challenge: the difficulty of accurately parameterizing patient-specific cardiovascular properties. In this work, we propose using Physics-Informed Neural Networks (PINNs) of 0D Windkessel models to uncover arterial stiffness and peripheral resistance from non-invasive measurements. By combining physical modeling and machine learning, this methodology solves inverse problems that recover critical mechanical parameters. Initial tests with simplified arterial configurations demonstrated the ability of PINNs to accurately predict vessel compliance, stiffness, and resistance, achieving errors below 2% in 7-vessel configuration, which opens the door for more ambitious tests. These findings indicate the potential of PINNs to augment direct numerical simulations by providing essential parameters whose measurement is difficult, expensive or intrusive.

1. Introduction

Personalized bloodflow modeling of the human cardiovascular system is a very useful tool for scientist and clinicians alike [1], offering critical information, reducing the need for invasive measurements or helping make informed decisions that can improve patient outcomes. To this end, three-dimensional (3D) [2] formulations, reduced models in one dimension (1D) [3] or simple aggregated (0D) models are used. These range from more detailed and computationally expensive, to simpler and faster to run. Particularly, 1D models for the arterial and venous tree coupled to aggregated models have proved to be an excellent choice balancing performance and computational cost [4]. Currently, a limitation of these models is the need to feed them with parameters representing the mechanical and topological characteristics of a specific patient's cardiovascular system.

In this paper, we propose a novel method based on machine learning techniques to recover the values of the mechanical parameters of the arterial tree using non-invasive measurements and a simple 0D physical model. Specifically, we show that Physics-Informed Neural Networks (PINNs), a type of neural network that incorporates a physical model in the form of a system of differential equations in its training, can be used successfully in the calibration [5]. PINNs are able solve the system directly, albeit less efficiently than traditional numerical methods, recovering the solution functions from the input variables, but more importantly, they can likewise be used to recover some unknown parameter of the system [6]. This constitutes solving an inverse problem, for which we consider PINNs to be a very competitive alternative to other methods.

The physical models where we have tested this approach are 0D models of arterial networks containing 3 and 7 vessels. This choice has been made in order to simplify the problem at hand and to reduce the number of parameters in the system.

2. Methodology

2.1. Aggregated model for networks of arteries.

A network of arteries is represented using an aggregated model by assuming only time-dependence. Thus, each vessel i is described by its average flow $Q_i(t)$, its average volume $V_i(t)$ and its pressure difference $\Delta p_i(t)$. We also consider each vessel to have a mechanical resistance to flow R_i ,causing pressure to drop, a mechanical compliance C_i allowing its volume to vary with pressure, and a certain inertance L_i given by the properties of the fluid. These parameters summarize the mechanical properties of each vessel and are computed as:

$$C = C_o\sqrt{\alpha} = \frac{V_o}{K}\sqrt{\alpha} \quad (1)$$

$$R = R_o\alpha^{-\frac{5}{2}} = \frac{\pi\beta\mu l}{V_o}\alpha^{-\frac{5}{2}} \quad (2)$$

$$L = L_o\alpha^{-1} = \frac{\rho l}{V_o}\alpha^{-1} \quad (3)$$

where $\alpha = V/V_o$ is the ratio between the volume and the reference volume of a vessel, K is the arterial stiffness, which depends on wall thickness and material properties and is closely linked with pulse wave propagation. Additionally, ρ and μ are density and viscosity of blood, respectively, l is the length of the vessel and β is a constant related to the velocity profile of the flow. Thus, following conservation of momentum, the difference of pressure in a vessel can be written as:

$$\Delta p_i = \left(R_i + R_{out,i}\right)Q_i + L_i\frac{dQ_i}{dt}R_i \quad (4)$$

where $R_{out,i}$ is the resistance of the vascular bed after a vessel and might be a parameter that needs to be found.
Additionally, at a vessel junction, where one parent vessel bifurcates into two children vessels, it is assumed that the pressure for the children vessels at the junction is the same. Thus, conservation of mass leads to the junction equation:

$$\left(C_2 + C_3\right)\frac{dp_1}{dt} = Q_1 - Q_2 - Q_3. \quad (5)$$

These ordinary differential equations form the system which is built into the PINN. However, a further closure relation is needed in order to relate pressure and volume:

$$p = p_o + K\left(\sqrt{\frac{V}{V_o}} - 1\right), \quad (6)$$

where p_o is the pressure at reference volume V_o.
This 0D model can be summarized using a circuit analogy, identifying flow with intensity and pressure with tension. Thus, a junction of three vessels can be represented as pictured in Figure 1.

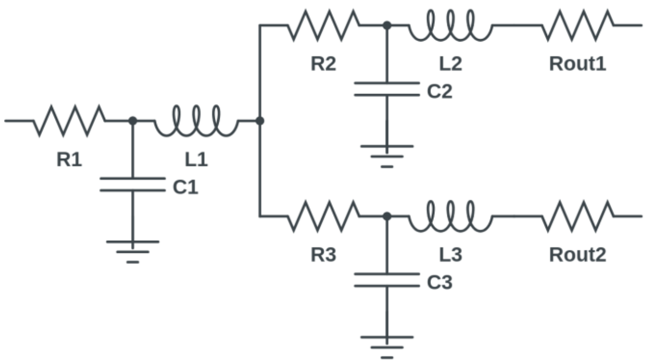

Figure 1: Representation of a 3-vessel junction using a circuit analogy.

2.2. Physically Informed Neural Networks

A PINN is a type of deep neural network (DNN) that includes information from a physical model in its training. Its architecture is identical to that of traditional neural networks, as it is built from layers of neurons, also called perceptrons. Each of these neurons receives an input vector $\boldsymbol{x}$ built from the output of the neurons of the previous layer and produces its own output $f(\boldsymbol{x})$ using a set of weights organized in matrix $\boldsymbol{W}$ and of biases $\boldsymbol{b}$. To control the behavior of each layer, an activation function σ is chosen, typically the hyperbolic tangent.
When solving the forward problem, all mechanical parameters are known a priori and the neural network takes one input in its first layer, the time variable, produces a pair of pressures and discharges for each vessel. Figure 2 shows a diagram of this type of network. As the outputs of the network have been produced by known chains of additions and multiplications, its exact derivatives with respect of the input variable can be computed, which allows in turn to compute the residual related with the system of ordinary differential equations. Some data, such as the pressure function in one of the vessels, is needed to train the PINN, which is done by minimizing the total loss using a gradient descent algorithm damped by a learning rate γ.

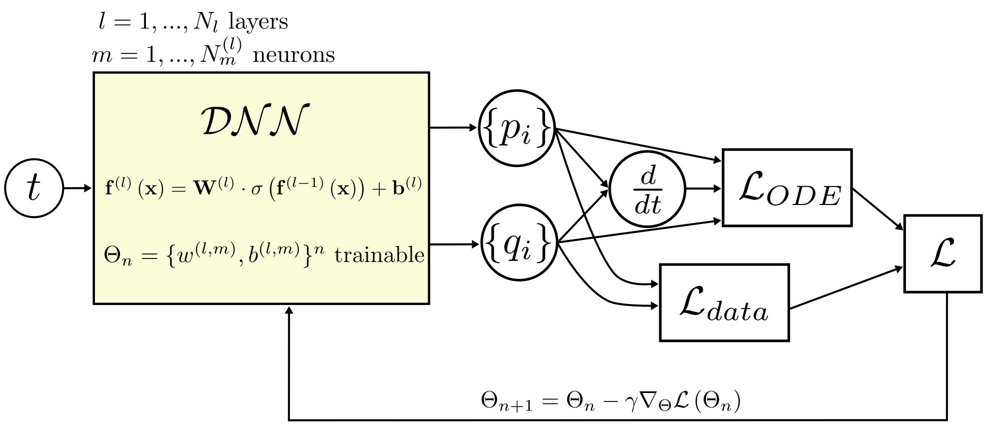

Figure 2: Diagram of the PINN for the direct problem.

This architecture can be altered to solve the inverse problem in which some parameters are unknown. Here, some additional data is needed, and the last layer of the deep neural network is altered to yield the extra outputs desired. A diagram is pictured in Figure 3.

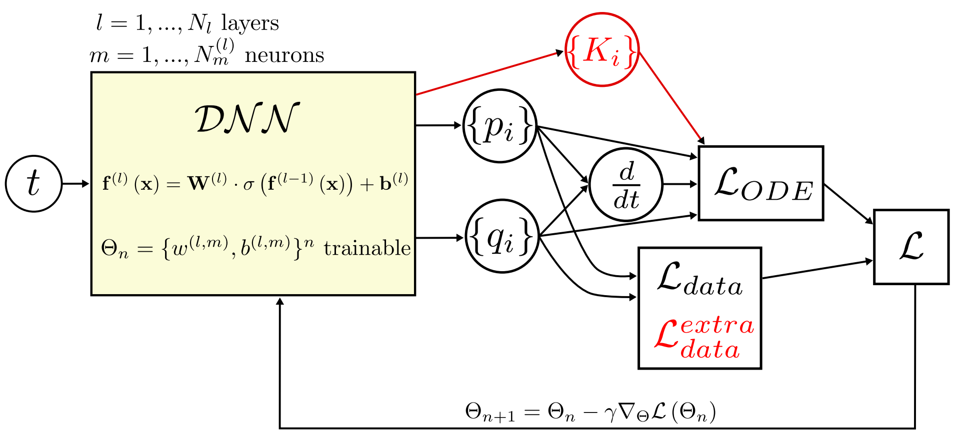

Figure 3: Diagram of the PINN for the inverse problem.

3. Results

3.1. Forward problem

Initially, we tested this methodology with a simple system consisting on a junction of three vessel whose properties are known and given by Table 1.

	l [cm]	$\bar{r}$ [cm]	c_o [cm/s]	p_o [mmHg]	p_{out} [mmHg]	R_{out} [mmHg/(cm s)]
Vessel 1	6.58	1.2675	456.0	75.0	-	-
Vessel 2	0.57	1.205	456.0	75.0	33.0	5.0
Vessel 3	4.2	0.6767	456.0	75.0	33.0	5.0

Table 1: Geometrical and mechanical properties of the case tested.

Only the discharge at vessel 1 was given, and the PINN was trained to find pressure and discharge in all three vessels. A comparison of the solution obtained by solving the system using a traditional method and using a PINN is pictured in Figure 4. The training lasted for several minutes, compared with the mere seconds that the traditional method takes.

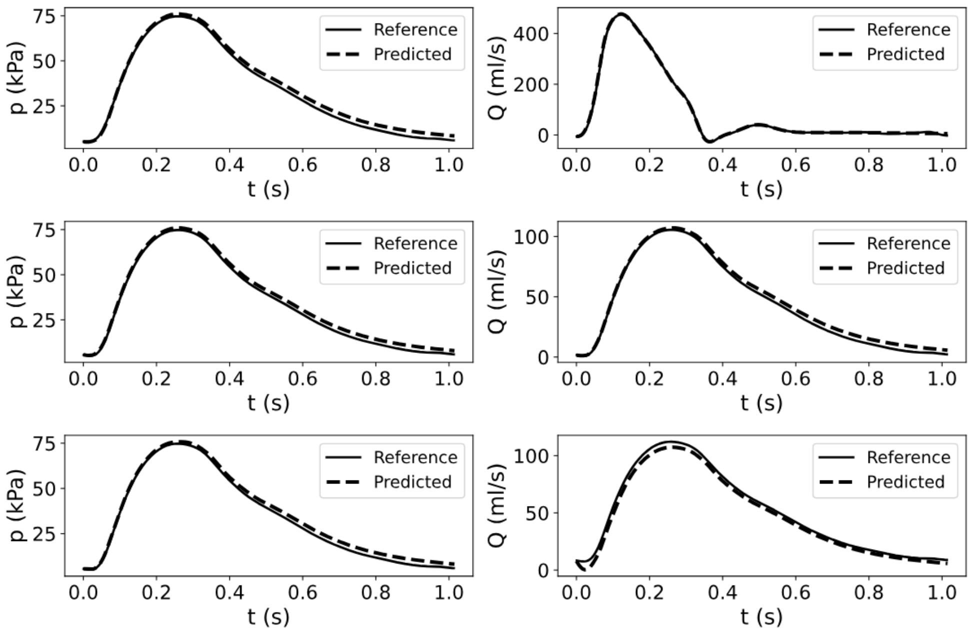

Figure 4: Results for the forward problem. Top row: vessel 1. Middle row: vessel 2. Bottom row: vessel 3.

3.2. Inverse problem

The same 3-vessel configuration is used to test the ability of the PINN to recover values of vessel stiffness and resistance at the outlets when these are hidden. Table 2 shows the relative errors in the prediction:

Table 2. Prediction error for vessel stiffness and outlet resistance at the 3-vessel case.

	ϵ_K %	$\epsilon_{R_{out}}$ %
Vessel 1	0.07	-
Vessel 2	0.08	0.8
Vessel 3	0.08	4.9

A more ambitious case was tested, in which 7 similar vessels were organized in a configuration with 3 Y-shaped junctions. Table 3 shows the relative error matrix in this case.

Table 3. Prediction error for vessel stiffness and outlet resistance at the7-vessel case.

	ϵ_K %	$\epsilon_{R_{out}}$ %
Vessel 1	1.45	-
Vessel 2	1.71	-
Vessel 3	1.66	-
Vessel 4	1.98	2.96
Vessel 5	1.98	1.79
Vessel 6	1.40	3.63
Vessel 7	1.40	1.84

Even in this more complex case, the PINN method is able to predict vessel stiffness keeping the error within less than 2%, while the outlet resistance is predicted with errors around 3%.

4. Discussion

We have shown the potential of PINNs built using aggregated models to retrieve unknown parameters in modeling arterial flow. This method is not competitive against direct numerical simulation when solving forward in time. However, it allows solving inverse problems, which cannot be done with forward methods. The good results obtained in simple test cases open the door to using this techniques in realistic cases of arterial trees.

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